Questions (8 pts)

Explain in a few sentences:

1) How to make a private communication with semiconductor laser's chaos?
2) What is a passively mode-locked laser?
3) What are the main differences between external and direct modulation?

Exercice (12 pts)

Interband cascade lasers have been invented by Rui Q. Yang from the University of Oklahoma. ICLs have many applications in gas sensing, because many molecules such as carbon dioxide and methane have strong rotational-vibrational absorption lines in the mid-infrared spectral range. Another application of ICLs is the free-space optical communication, thanks to the transmission window of atmosphere in 3–5 microns. As shown in Figure 1, the active region is composed of many stage cascade periods of quantum wells. Therefore, the emission is based on interband transition (like in quantum well laser diodes) wherein one electron (hole) recombines with m holes (electrons) and hence produces a total number of m photons.

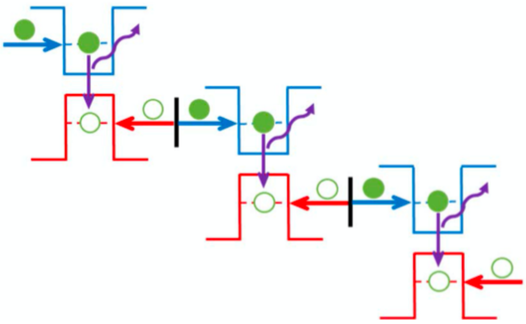

Fig .1: Schematic illustrating the carrier injection and photon emission processes in the ICL active region. Closed dots stand for electrons and open dots stand for holes. Photon emission is indicated with purple arrows.

The corpuscular equations describing the ICL are the following

$$\frac{dN}{dt} = \eta\frac{I}{q} - G_N(N - N_t)S - \frac{N}{\tau_c}$$

$$\frac{dS}{dt} = mG_N(N - N_t)S - \frac{S}{\tau_p} + m\beta\frac{N}{\tau_c}$$

where N is the carrier number per gain stage, S the photon number in the laser cavity, I the pump current, η is the current injection efficiency, N_t the transparent carrier number, β the spontaneous emission factor, m the cascade stage number, and τ_p the cavity photon lifetime. The gain coefficient G_N is the differential gain while τ_c is the carrier recombination lifetime.

1) Show that the output power below threshold is linked to the spontaneous emission factor. Plot S-I.
2) Determine the threshold carrier density N_{th} and threshold current I_{th}.
3) Above threshold, prove that $S = m\eta\frac{\tau_p}{q}(I - I_{th})$. Plot S-I.
4) The evolution matrix of the ICL gives insights on rates of evolution of each state variable such as photon and carrier populations. By performing a linear stability analysis of the steady-state conditions, the evolution matrix stands as follows,

$$J = \begin{pmatrix} -\frac{1}{\tau_c} - G_N S & -\frac{1}{m\tau_p} \\ mG_N S & 0 \end{pmatrix}$$

5) Determine the characteristic polynomial.
6) Show that the eigenvalues are

$$\lambda_{\pm} = -\Gamma \pm \sqrt{\Gamma^2 - \omega_r^2}$$

Express Γ and ω_r as a function of G_N, τ_c, τ_p and S. What do they correspond to?

7) Assuming $\Gamma << \omega_r$ show that the eigenvalues are complex numbers.
8) Figure 2(a) shows the modulation response at various pump currents for m=5. Explain the origin of the peak. Give the maximum 3-dB bandwidth achievable with this ICL. Verify that the 3-dB bandwidth frequency follows the relationship $f_{3dB} = 1.55 \times f_r$.
9) Figure 2(b) displays the 3-dB bandwidth as a function of the normalized pump current. Using 6) explain the role of the cascade stage number *m* on the modulation bandwidth.

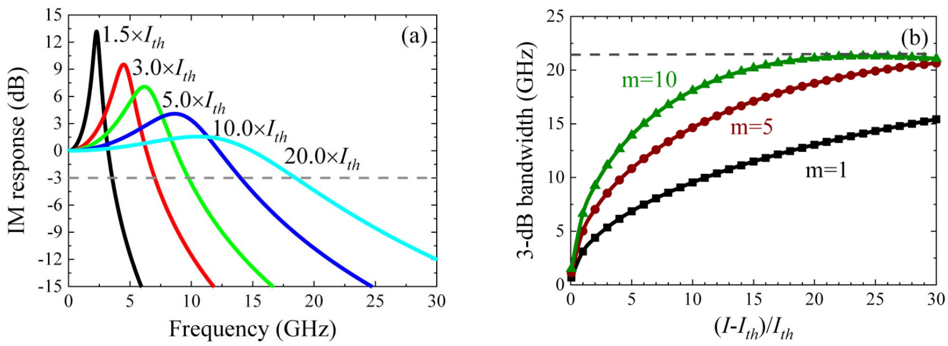

Fig .2 (a) Modulation response at various pump currents for m=5. The dashed line indicates the 3-dB bandwidth. (b) 3-dB bandwidth as a function of normalized pump current. The dashed line indicates the maximum achievable 3-dB bandwidth.